FUSION PLASMA PHYSICS

INTERPRETATION AND CONTROL OF HELICAL PERTURBATIONS IN TOKAMAKS

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During the period January-December 2005, the theoretical and modeling research activity of the "**Mathematical Modelling for Fusion Plasmas Group**" of the National Institute for Laser, Plasma and Radiation Physics (NILPRP), Magurele - Bucharest, Romania has been focalized on:

- ***Plasma models for feedback control of helical perturbations***, activity performed in collaboration with the Max-Planck - Institut für Plasmaphysik (IPP), Tokamakphysics Department, Garching, Germany, and represents a continuation of our activity from 2004.

1. Introduction

It is known that the maximum achievable β in "advanced tokamaks" is limited by the pressure gradient driven ideal external-kink modes (EKM), evolving on a fast time scale of 10^{-6}s. When a tokamak plasma is surrounded by a close fitting resistive wall, the relatively fast growing ideal EKM is converted into the far more slowly growing "resistive wall mode" (RWM) which grows on the characteristic τ_w= L/R time of the wall and has virtually identical stability boundaries to those of the EKM in the complete absence of a wall.

The "advanced tokamak" concept is economically attractive if the ideal external kink beta limit is raised substantially. This is possible only if the RWM is stabilized by two factors: passive (close fitting conducting wall) and active (feedback coils) stabilization and rapid plasma rotation. A plausible stabilization mechanism is a combined effect of plasma rotational inertia and dissipation due to interaction with the sound wave continuum at a toroidally coupled resonant surface lying within the plasma [1, 2]. In the latest RWM theories [3, 4], plasma dissipation is associated with internal Alfvén resonances. Fitzpatrick and Aydemir [5, 6] have developed a simplified mechanism where the plasma dissipation is provided by the edge plasma viscosity. According to all models, the critical toroidal plasma velocity required to stabilize the RWM is of order $(k_{\parallel}a)V_A$ ($k_{\parallel}$ is the parallel wave vector and V_A is the Alfvén plasma velocity). There is, at present, no clear consensus of opinion as to what are the necessary ingredients for the stabilization of this mode.

It is to note that the stabilization of RWM in ITER, where it is probably not possible to maintain a very fast plasma rotation is still an open problem.

2. Results

The goal of our common research was to advance the physics understanding of the RWM stability, including the dependence on plasma rotation, wall/plasma distance, and active feedback control, with the ultimate goal of achieving sustained operation at beta values close to the ideal-wall beta limit through passive or/and active stabilization of the RWM. With this in view, the aim of our work was to find an optimal feedback system needed for stabilizing resistive wall mode instabilities in tokamak plasma.

Up to now, we have developed an analytical model, based essentially on Fitzpatrick's resistive shell mode model [5, 6]. We have considered a large aspect ratio, low beta, circular flux surface tokamak equilibrium; linearised, incompressible, non-ideal (i.e. including the effects of plasma inertia, resistivity and viscosity) equations of reduced MHD; a general m/n current-driven external-kink mode in the presence of plasma rotation and a thin resistive non-uniform vacuum vessel with feedback and detector coils placed around the tokamak plasma. The plasma itself is considered as formed of: the main, current carrying plasma, the region between the edge of the main plasma and the vacuum vessel, an inertial layer in the outer regions of the current carrying plasma. The influence of different parameters on the growth rate of the mode has been investigated.

The following **milestones** have been considered in 2005:

- investigation of the influence of the position, dimensions and resistivity of the feedback system on the growth rate of the RWM;
- investigation of the influence of the plasma rotation relative to the shell on the behaviour of the RWM in the presence of the feedback system;
- calculation of the minimum rotation of the plasma inertial layer required for the stabilization of the RWM;
- study of the influence of the anomal viscosity as dissipation mechanism of the plasma rotation on the stabilization of the RWM;
- description of the influence of the cumulative effect of plasma rotation and coupling between RWM and eddy currents induced in the passive shell surrounding the plasma;
- calculation of the coupling effect between the RWM and adjacent modes on the RWM growth rate and its rotation;
- extension of the expression of the potential energy due to a plasma displacement for tearing modes in diverted geometries to the external kink modes;
- to calculate the surface current corresponding to a given plasma mode;
- calculation of the patterns of the induced eddy currents for different plasma parameters.

Observation: *in agreement with our German partners, we have concluded that it is of great interest to continue and develop the analytical model in order to obtain physical results which could not be obtained by a 2D (or 3D) numerical simulation and to postpone the numerical model. Therefore, the three milestones written with italic characters, have been replaced with the following three ones:*

- elaboration of an improved 2D analytical model that describes the behavior of the resistive wall modes, by identifying and elimination of the spurious roots of the dispersion relation;
- elaboration of a numerical code with a "friendly" interface for users - for RWM calculation;
- derivation of a new and improved dispersion equation concerning both the growth rate and the real rotation frequency of the resistive wall modes.

The following system has been considered [7-11]: the active part of the feedback system consists of pairs feedback coils – detector coils and by its radial position to the plasma, its dimensions and by the level of amplification fed into the feedback coils the behavior of the RWM can be modified. The detectors position between the shell and the feedback coils have to play an important role in stabilizing the RWM (Figure 1).

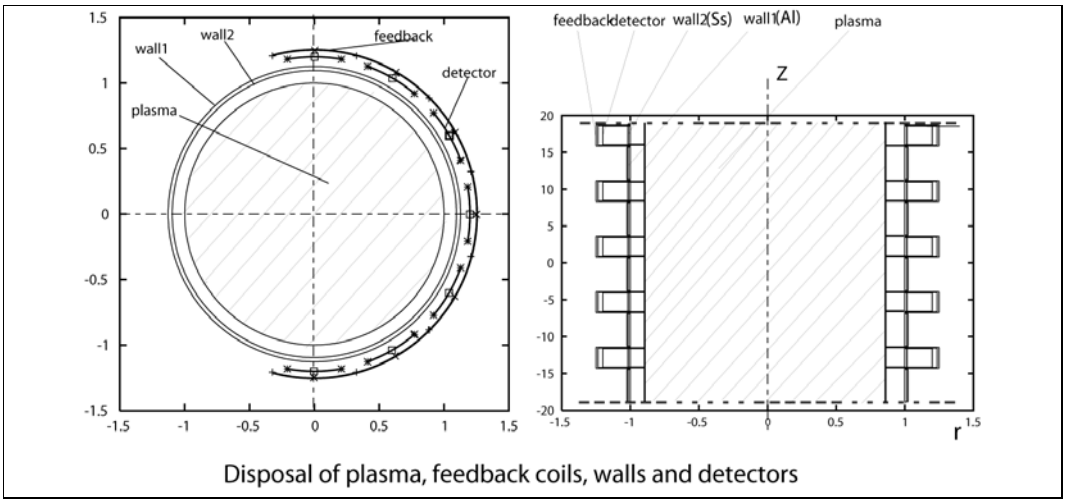

Figure 1. The considered plasma, wall, feedback coils and detectors system

The standard plasma and feedback system parameters are given in Table 1.

In a "cylindrical" tokamak, periodic in the z direction with periodicity length $2\pi R_0$, having a large aspect ratio and low β ordering scheme and a Wesson like current density distribution [12], the linear stability equations can be obtained by considering in a standard right-handed cylindrical polar co-ordinates (r, θ, z), the linearized force balance equation (viscous force), the linearized Ohms law and assuming for the perturbed quantities a dependence of the form $\exp[i(m\theta - nz/R_0)]$. Expressing the perturbed magnetic field in terms of a poloidal flux function and the velocity in terms of a displacement stream function (assuming here an $\exp(\gamma_0 t)$ time dependence), one obtains the linearized reduced-MHD equations. By particularizing these equations for the ideal-MHD region (bulk plasma) one define the stability control parameter of the external kink mode (the logarithmic derivative of the perturbed flux function) and the edge region (layer equations) taking into account that the edge plasma presents a rotation ($\gamma_0=\gamma+i\omega_0$, $\omega_0=m\Omega_\theta-n\Omega_z$). Finally, considering: - the jump conditions at the wall, - the jump of ψ across the feedback coils, - the feedback current stream function, - the inductive voltage generated in the feedback coils, - the inductive voltage generated in the detector loop, - the circuit equation, - the applied feedback voltage, related to the voltage measured by the detector loop, via the proportional and derivative gains, the full dispersion relation reads like

$$\sum_{j.k}\left(\gamma_0^2 q_{jk2}^{mn}+\gamma_0 q_{jk1}^{mn}+q_{jk0}^{mn}\right)\left(\gamma_0^4 A_{jk4}^{mn}+\gamma_0^3 A_{jk3}^{mn}+\gamma_0^2 A_{jk2}^{mn}+\gamma_0 A_{jk1}^{mn}+A_{jk0}^{mn}\right)\frac{1}{k}K_{wf}^{jk}\left(Q_{wf}^{jk}\right)^2\Psi_w^{jk}=0$$

The second order polynomial appearing in the above dispersion relation represents spurious roots that do not characterize the RWM of the considered

Table 1. Standard plasma parameters for RWMs investigation

Parameters	Value	Units	Observations
a	1	m	small plasma radius
R_0	6	m	equivalent big plasma radius
q_0	1.3		safety factor at plasma center
q_a	2.9		safety factor at plasma boundary
B_z	2.1	T	toroidal magnetic field
$\mu\perp$	$9*10^{-14}$	kg/m/s	anomalous viscosity at a=1
r_w	1.15	m	wall radius
τ_A	$3*10^{-7}$	s	Alfvén time-scale (edge plasma)
τ_R	1.4	s	resistive time-scale (edge plasma)
τ_V	10^5	s	viscous time-scale (edge plasma)
$d_{w\,Al}$	0.01	m	aluminium wall thickness
$d_{w\,SS}$	0.002	m	stainless steel wall thickness
ρ_{Al}	$2.7*10^{-8}$	Ωm	aluminium resistivity
ρ_{SS}	$7.2*10^{-7}$	Ωm	stainless steel resistivity
r_f	1.25	m	feedback coils radius
d_f	0.005	m	feedback coils thickness
ρ_f	$1.6*10^{-8}$	Ωm	feedback coils resistivity
r_d	1.2	m	detector radius
M/N	4/5		no. of f. coils/ Al & Ss pieces (pol./tor.)
ω_θ	0	rot/s	poloidal plasma rotation
ω_φ	-30000	rot/s	toroidal plasma rotation
G_p	31	V/V	proportional gain $V_c=G_d*dF_d/dt + G_pF_d$
G_d	5.5	V/Wb	differential gain
m_0/n_0	3/1		considered RWM
$a_{m0/no}$	1.0171	m	resonance radius
n_θ	7		no. feedback & detector coils In poloidal direction

plasma. $\gamma_0 = \gamma + i\omega_0$ represents the variation rate of the RWM and the rotation velocity of the edge plasma, Ψ is the perturbed flux function, while the parameters enclosed in the equation contains all information of the plasma and the system of coils and detectors used for the feedback of the RWMs. The summation takes place over the poloidal and toroidal plasma modes taken into account.

Starting from the "standard" parameters presented in Table 1, the influence of different parameters on the growth rate of the mode has been investigated.

A very important issue to achieve in tokamak plasmas is a higher plasma normalized beta while preserving stabilization for the edge plasma modes. We have shown the growth rate dependence on the stability parameter $\kappa = (\beta_N-\beta_{\text{no-wall}})/(\beta_N-\beta_{\text{ideal-wall}})$ (κ =0, corresponds to the n-wall plasma case, while κ =1 to the ideal-wall plasma case) with and without feedback and for different locations of the passive shell, feedback and detectors coils. Practically, use has been made of an equivalent parameter: $\kappa = (r_w^{2m}- a^{2m})/(r_c^{2m}- a^{2m})$, where the critical radius r_c defines the radius up to which the shell converts the ideal external kink mode into a non-rotating RWM (r_c=1.381m) and m is the poloidal wave number of the considered RWM. Figures 2 (a) and (b) show the appropriate way for placing the feedback system around the plasma. Placing the feedback coils and detectors between plasma and the passive shell seems to be the best choice for RWM stabilization and, consequently, for obtaining a better plasma stability parameter. The obtained result could be relevant for ITER.

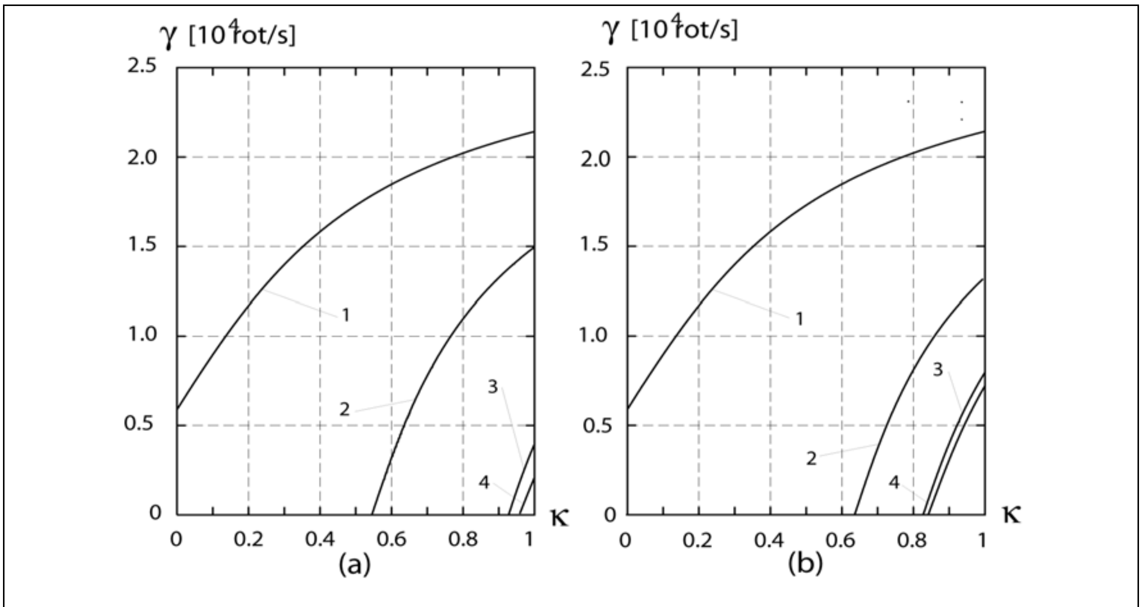

Figure 2. $\gamma(\kappa)$ dependence, with $\kappa=(\beta_N-\beta_{\text{no-wall}})/(\beta_N-\beta_{\text{ideal-wall}})$, for G_p= 5.5, G_d= 31, Ω_φ= -300 rot/s and d_{w1}= d_{w2}= 0.0001m, in two cases: ***a)*** *(1) without feedback, (2) $r_f - r_w$ = 0.1m and $r_d - r_w$ = 0.05m, (3) $r_f - r_w$ = 0.1m and $r_w - r_d$ = 0.05m, (4) $r_w - r_f = r_w - r_d$ = 0.05m;* ***b)*** *(1) without feedback, (2) $r_f - r_w$ = 0.05m and $r_d - r_w$ = 0.025m, (3) $r_f - r_w$ = 0.05m and $r_w - r_d$ = 0.025m, (4) $r_w - r_f = r_w - r_d$ = 0.025m, where, r_w, r_d, r_f are the radius of the wall, detector coils and feedback coils, respectively.*

Figure 3 shows the growth rate of the 3/1 RWM calculated as a function of the plasma toroidal rotation for various wall radii in the absence of feedback. A wall that is close to the plasma gives rise to a RWM with a real frequency growth rate (the imaginary part of γ_0) closer to zero. We found again that initially the plasma rotation has a destabilizing effect on the RWM and that only when γ_{0i} is greater than the real one γ_{0r}, does the stabilization of the mode start. According to Figure 3, somewhat counter intuitively, the optimum configuration is to place the passive shell as far away from the plasma as is consistent with the stabilization of the external-kink instability. The descending rate of γ_0 is lower for a close-fitting passive shell in the presence of a strong edge plasma rotation. The reason for this fact consists in an easier mode decoupling from the shell in the case of a far-fitting resistive shell. Also, it can be seen an initially destabilizing effect of the edge plasma rotation on the RWM; as long as the shell is able to lock the mode, the angular rotation "feeds" energetically the RWM which grows radially. When a sufficient angular rotation is provided, it unlocks the mode from the shell, the RWM starting to stabilize itself.

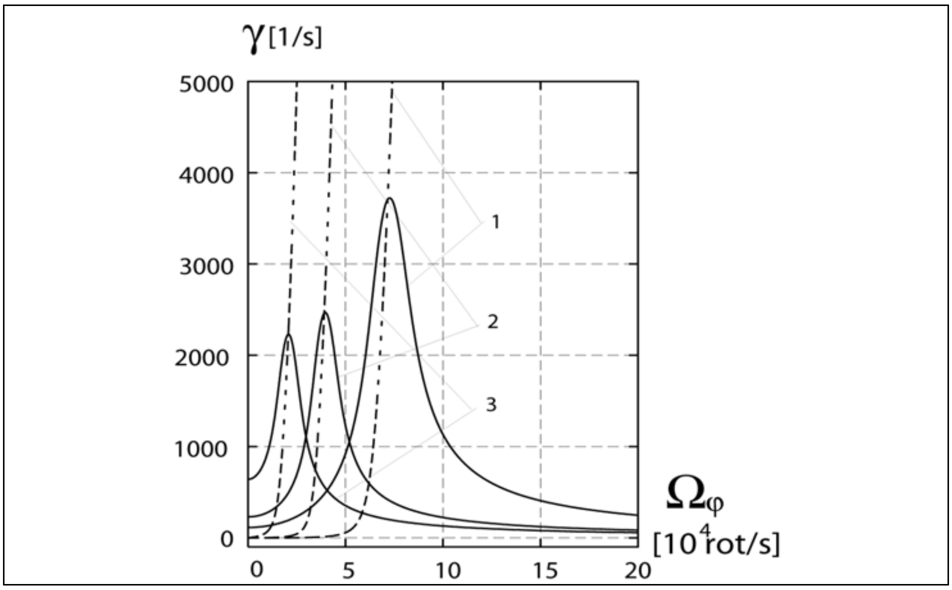

Figure 3. γ (Ω_φ) dependence without feedback and for three different wall radii: (1) r_w= 1.1m, (2) r_w=1.2m and (3) r_w=1.3m. The solid lines represent the real part of γ, the growth rate, while the doted lines the imaginary part, the rotation of the mode.

The growth rate dependence on the location of the resistive wall r_w, by keeping the positions of both detector and feedback coils, are presented in Figure 4. The destabilizing effect by approaching the wall radius to the critical Newcomb radius [13] with no rotation has been found. If the wall radius is placed above this radius, the mode cannot be stabilized.

The dependence of the plasma normalized beta as function of the toroidal angular velocity of the plasma edge for different values of the anomalous perpendicular viscosity coefficient in the plasma (μ_1= 9x 10^{-14}kg/m/s, μ_2= 4.5x 10^{-13}kg/m/s, μ_3= 9x 10^{-13}kg/m/s) is reported in Figure 5. It can be seen that the area of the stable region for the resistive wall mode is an increasing function of dissipation by viscosity in the plasma. At the same time, the minimum edge rotation required for the stabilization decreases as the dissipation takes higher values. κ is plotted between the no-wall plasma beta (corresponding to κ = 0) and ideal-wall plasma beta (corresponding to κ = 1).

In Figure 6 the growth rate dependence of derivative gain factor Gd is given for different wall materials. Similar results were obtained for dependence with

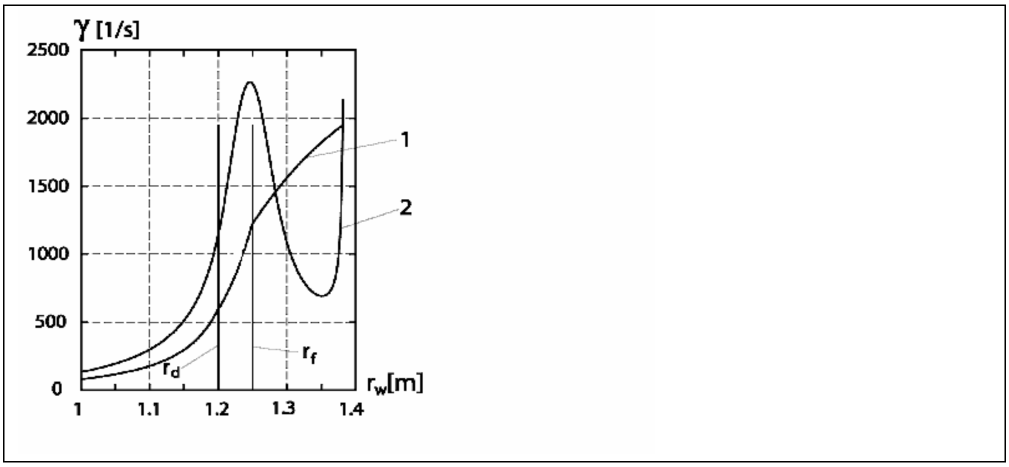

Figure 4. γ (r_w) dependence with r_d=const. and r_f=const. (vertical solid lines). G_d=31 and G_p=5.5; (2) without feedback.

the proportional gain factor Gp, both investigations showing that the best disposal of the alternate wall pieces is pieces with aluminium without active

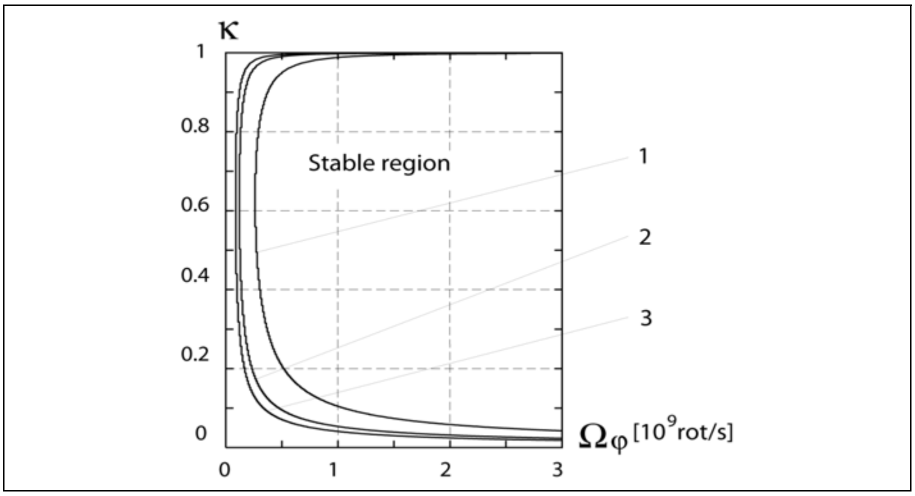

Figure 5. Stability parameter κ dependence on toroidal plasma rotation Ω_φ for different perpendicular plasma edge viscosities $\mu_\perp$. (1) $\mu_\perp = 9X10^{-14}$ kg/m/s; (2) $\mu_\perp = 4.5X10^{-13}$ kg/m/s; (3) $\mu_\perp$=$9X10^{-13}$ kg/m/s

feedback system and stainless steel pieces having feedback coils and detectors.

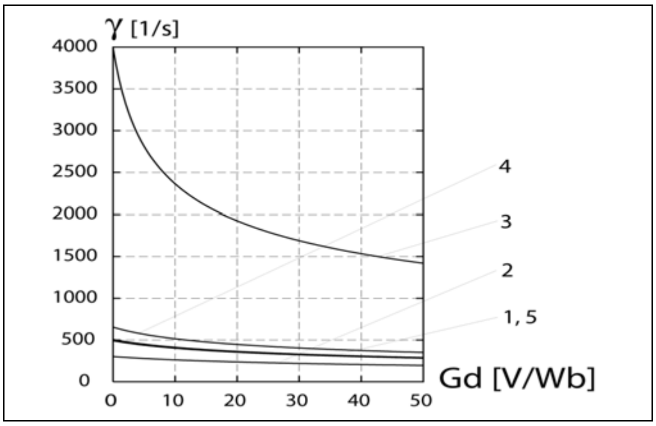

Figure 6. γ dependence on derivative gain factor G_d for G_p=0 and different wall resistivities. (1)) $\eta_1=\eta_{Al}$, $\eta_2=\eta_{ss}$, (2)) $\eta_1=\eta_{Al}$, $\eta_2=\eta_{Al}$, (3)) $\eta_1=\eta_{ss}$, $\eta_2=\eta_{ss}$, (4)) $\eta_1=\eta_{ss}$, $\eta_2=\eta_{Al}$, (5)) $\eta_1=\eta_{Al}$, $\eta_2=\eta_{ss}\times 10$

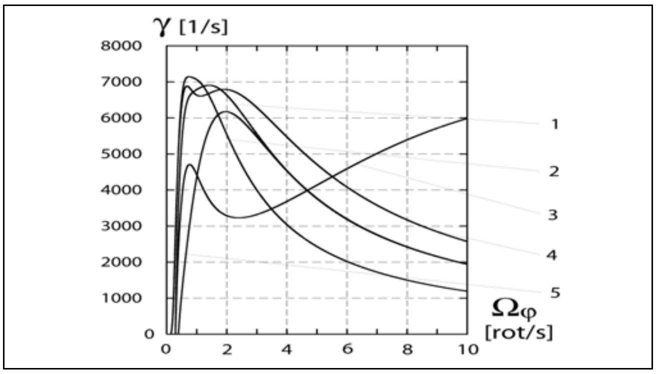

Figure 7. Growth rate γ_r dependence on toroidal plasma rotation Ω_φ with feedback and for different wall resistivities: (1) $\eta_1=\eta_{Al}$, $\eta_2=\eta_{ss}$, $G_p=5.5$, $G_d=31$, (2) $\eta_1=\eta_{Al}$, $\eta_2=\eta_{Al}$, $G_p=5.5$, $G_d=31$, (3) $\eta_1=\eta_{ss}$, $\eta_2=\eta_{ss}$, $G_p = 5.5$, $G_d = 31$, (4) $\eta_1=\eta_{ss}$, $\eta_2=\eta_{Al}$, $G_p = 5.5$, $G_d = 31$, (5) $\eta_1=\eta_{Al}$, $\eta_2=\eta_{ss}$, $G_p=55$, $G_d=310$. $\eta_{Al}=0.465X10^{-07}$ Ωm and $\eta_{ss}=0.9X10^{-06}$Ωm.

3. Conclusions

- in order to understand the physics of resistive wall mode stability, the Fitzpatrick model has been considered and generalized;
- we sought to find the necessary rotation which, combined with an appropriate distance passive shell - plasma surface and active feedback, can decouple the kink modes from the eddy currents in the shell, leading to the stabilization of the RWMs;
- an important destabilizing factor of the RWMs we have investigated was the coupling between a RWM and its adjacent modes. The inhomogeneity of the passive shell and the presence of the discrete active feedback system are coupling the plasma modes in such a way that the RWM becomes more unstable;
- another result we have obtained was that the angular extent of the feedback coils is important in order to stabilize the RWM;
- considering the alternative disposal of aluminum and stainless steel wall pieces, we found that the best combination of wall pieces is that where the stainless steel pieces only are provided with a feedback system , the aluminum wall pieces fulfilling a passive feedback role only;
- by decreasing the degree of the polynomial dispersion relation, we were able to improve the accuracy of our results;
- finally, a numerical interactive code for the semi-analytical model of RWMs calculation in the form of a “handbook” with “friendly” interface for users has been elaborated.

4. Next steps

We intend to continue the investigation of the resistive wall modes in tokamaks, by considering the following **milestones**:

- calculation of the coupling effect between the RWM and adjacent modes on the RWM growth rate and its rotation;
- to estimate the dissipation at the sideband rational surfaces (i.e. other than the intrinsically unstable principal harmonic).

- investigation of the coupling of different poloidal harmonics due to the non-sinusoidal nature of the feedback currents;
- investigation of the low and high dissipation stability boundaries concerning the RWM.
- finding the optimal design of the active feedback system for the control of the resistive wall modes.
- Developing of a 2D numerical algorithm for calculation of resistive wall modes in a tokamak configuration with axisymmetric geometry (equations and algorithm only)

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5. Collaborative actions

Project: *INTERPRETATION AND CONTROL OF HELICAL PERTURBATIONS IN TOKAMAKS*

Partner: Tokamakphysics Department of the Max-Planck-Institut für Plasmaphysics (IPP), Garching, Germany

Objective: Plasma models for feedback control of helical perturbations

Milestone: Developing of a 2D semi-analytical code for calculation of resistive wall modes

The objective of this common research was to advance the physics understanding of RWMs stability, including the dependence on plasma rotation, wall/plasma distance, and active feedback control, with the ultimate goal of achieving sustained operation at beta values close to the ideal-wall beta limit through passive or/and active stabilization of the RWMs.

Conceived to be a "handbook" for RWMs, the case of a large aspect-ratio, low beta, circular cross-section, rotating, viscous tokamak plasma surrounded by a thin, non-uniform resistive shell and non-uniform feedback coils has been considered for this code.

For given plasma parameters like small radius, plasma current, plasma current density profile, poloidal beta, and toroidal magnetic field, the code calculates the growth rate of the RWMs perturbation under the influence of : - a periodic or non-periodic shape and position of the resistive shell made from different conducting materials, - a periodic or non-periodic shape, position and distribution of the feedback coils, - a periodic or non-periodic shape, position and distribution of the detectors coils, - a dissipation mechanism via the plasma viscosity, and - coupling of different poloidal harmonics due to the non-sinusoidal nature of the feedback currents.